cients for ⑧×⑧ to obtain ratios of various physical scattering amplitudes:

$$a(KK)=a^{3/2}(K\pi)=a^{1/2}(K\pi)=a^2(\pi\pi)$$
$$=a(K\eta)=a(\pi\eta)=a^S, \quad (45)$$
$$a^1(\pi\pi)=0, \quad a^0(\pi\pi)=\tfrac{5}{2}a^S, \quad a(\eta\eta)=\tfrac{3}{2}a^S.$$

Equation (45) relates the a^S of Eqs. (37) and (43) with those for several other processes; in the present approach such relations are not universal but result from the specific assumption of a higher symmetry.

(g) Boson-Fermion Octet Scattering

Although certain objections may be raised[3] against the use of octet symmetry for fermion-boson interactions, we can apply the formulas without difficulty; in this case, only twofold symmetry obtains, and there are three possible independent cross sections in the s channel:

$$\begin{aligned} &a(\eta N)\\ &a^0(\bar{K}N)=\tfrac{1}{2}[3a^1(KN)-a^0(KN)],\\ &a^1(\bar{K}N)=\tfrac{1}{2}[a^1(KN)+a^0(KN)]\\ &\qquad =a^{3/2}(\pi N)=a^{1/2}(\pi N).\end{aligned} \quad (46)$$

The separate Eqs. (46) are sufficient to satisfy Pomeranchuk's theorem *without* necessarily implying isotopic spin independence for KN or $\bar{K}N$ scattering. In this case isotopic spin is only part of a more inclusive symmetry. The appearance in high-energy data of a familial relation between πN and KN scattering has been pointed out.[15]

ACKNOWLEDGMENT

The author wishes to thank Professor M. Kawaguchi for critical and stimulating discussions.

[15] R. Serber, Phys. Rev. Letters **13**, 32 (1964).

PHYSICAL REVIEW VOLUME 136, NUMBER 6B 21 DECEMBER 1964

Boson-Pole Model in K-Meson and η-Meson Decays*

K. TANAKA†
Argonne National Laboratory, Argonne, Illinois
(Received 31 July 1964)

The direct term of the decay mode $K\to\pi+\pi+\gamma$ and the decay amplitudes $K\to\pi+\pi+\pi$ and $\eta\to\pi+\pi+\pi$ are obtained on the basis of the boson-pole model to all orders in the strong interaction, provided the interaction (except for the weak vertex) is SU_3-invariant. The weak vertex is assumed to transform as K°. Then all $K\to\pi+\pi+\gamma$ modes, except $K_2^0\to\pi^+\pi^-\gamma$, and all $K\to\pi+\pi+\pi$ and $\eta\to\pi+\pi+\pi$ modes, except $(\eta|\pi^+\pi^-\pi^0)$, are shown to vanish. It is concluded that the boson-pole model together with unitary symmetry is untenable for $K\to 3\pi$ decay modes.

1. INTRODUCTION

THE boson-pole model[1] has been used to compute the direct term of the $K\to 2\pi+\gamma$ mode,[2] $K\to 3\pi$ and $\eta\to 3\pi$ modes,[3,4] as well as other processes. In this model, the initial boson P_1 converts into another boson P_2 by a weak vertex $P_1\to P_2$, which then turns into the final states $P_2\to P_3+P_4+\gamma$ (radiative mode) or $P_2\to P_3+P_4+P_5$ (3π mode) by electromagnetic and strong interactions, i.e., for example,

$$P_1\to P_2\to P_3+P_4+\gamma.$$

There is another class of diagrams in which the weak vertex follows the electromagnetic and strong interactions, i.e.,

$$P_1\to P_2+P_3+\gamma\to P_4+P_3+\gamma.$$

The part of the diagram that depends on the electromagnetic and strong interactions can be regarded in unitary space as $\gamma+P\to P+P$ and $P+P\to P+P$, i.e., two octets transforming into two other octets.[5,6] The photon transforms as a singlet ϕ^0 plus the third component of a vector ρ^0, i.e.,

$$\gamma=-\tfrac{1}{2}\phi^0-\tfrac{1}{2}\sqrt{3}\rho^0.$$

It will be shown later that because of conservation of momentum, it is a reasonable approximation to combine the two types of diagrams when the bosons have their respective physical masses.

* Work performed under the auspices of the U. S. Atomic Energy Commission.
† Present address: Ohio State University, Columbus, Ohio.
[1] G. Feldman, P. Matthews, and A. Salam, Phys. Rev. **121**, 302 (1961).
[2] S. V. Pepper and Y. Ueda (unpublished); Y. S. Kim and S. Oneda, Phys. Letters **8**, 83 (1964). The author would like to thank V. Barger and M. Kato for pointing out that there is another class of diagrams for $K^+\to\pi^++\pi^0+\gamma$ that cancel out the contributions obtained above.
[3] S. Hori, S. Oneda, S. Chiba, and A. Wakasa, Phys. Letters **5**, 339 (1963), and references given there.
[4] C. Kacser, Phys. Rev. **130**, 355 (1963), and references given there.
[5] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[6] We have explicitly verified that the amplitude of $8\times8\to8\times8$ lead to the same results as the amplitudes of $8\to8\times8\times8$. The former is more convenient to consider. The author benefited by discussions with R. G. Sachs and B. Barrett.

The $(\gamma P|PP)$ amplitudes are expressible in terms of the channel amplitudes A_{27}, A_{8s}, A_1, $A_{\overline{10}}$, A_{10}, A_{8a}, A_{as}, and A_{sa}. Charge-conjugation invariance requires $A_{27}=A_{8s}=A_1=A_{8a}=0$, and $A_{\overline{10}}=-A_{10}$ so there are three independent amplitudes.[7] The $(PP|PP)$ amplitudes are expressible in terms of five independent channel amplitudes B_{27}, B_{8s}, B_1, B_{10}, and B_{8a}.[8]

Therefore, all orders in the strong interactions, the lowest order in the electromagnetic interactions for $P\rightarrow P+P+\gamma$, and all orders in the strong interactions for $P\rightarrow P+P+P$ are taken into account. The decay modes of K and η are then calculated on the basis of unitary symmetry and almost all amplitudes are shown to vanish. Since these are rigorous consequences of the boson-pole model combined with unitary symmetry, one can test the validity of this approach.

The radiative decay modes of K and the 3π decay modes of K and η are considered in Sec. 2 and Sec. 3, respectively. In Sec. 4, the consequences of the results are discussed.

2. $K\rightarrow\pi+\pi+\gamma$

There are two mechanisms that can give rise to the process $K\rightarrow\pi+\pi+\gamma$. One is the inner bremsstrahlung term, in which the photon is emitted from the initial or final charged particle; the other is the direct term, in which the photon is emitted from the intermediate charged particle.

For the K decays, the direct term is obtained primarily by

$$K\rightarrow\pi\rightarrow\pi+\pi+\gamma \quad\text{and}\quad K\rightarrow K+\pi+\gamma\rightarrow\pi+\pi+\gamma.$$

The $P\rightarrow P+P+\gamma$ part of the decay amplitude results from the electromagnetic and strong interactions. In unitary space, it is expressed as $\gamma+P\rightarrow P+P$ whose channel amplitudes in SU_3 are listed in Table I. The diagrams in the boson-pole model that contribute to $(K^+|\pi^+\pi^0\gamma)$ are given in Fig. 1.

The structure of the diagrams consists of a weak vertex, a boson propagator $D=(M^2-m^2)^{-1}$, and the amplitude $(P|PP\gamma)$, where M and m are the masses of the K and π mesons, respectively. When the two-point vertex follows the amplitude $(P|PP\gamma)$, then the boson propagator is $(-D)$. We assume that the $|\Delta S|=1$, $\Delta Q=0$ vertex transforms as the K^0 meson in order to insure the usual $|\Delta I|=\frac{1}{2}$ rule, and further that it is

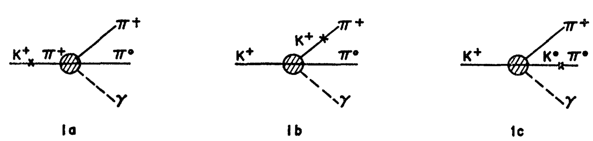

FIG. 1. Diagrams for the decay mode $K^+\rightarrow\pi^++\pi^0+\gamma$.

[7] This fact follows from $C|V)=-|\bar{V})$, $C|P)=|\bar{P})$, where V and P represent vector mesons and pseudoscalar mesons, respectively.

[8] K. Itabashi and K. Tanaka, Phys. Rev. **135**, B452 (1964).

TABLE I. The $\gamma+P\rightarrow P+P$ amplitudes in terms of channel amplitudes. The amplitudes have been multiplied by four.

	A_{10}	A_{as}	A_{sa}
$(\gamma\pi^+|\pi^+\pi^0)/\sqrt{3}$	$\frac{1}{3}$	$\cdots$	$-2/3\sqrt{5}$
$(\gamma K^+|K^+\pi^0)/\sqrt{3}$	$\frac{2}{3}$	$1/\sqrt{5}$	$-1/3\sqrt{5}$
$(\gamma K^+|\pi^+K^0)/\sqrt{6}$	$-\frac{1}{3}$	$-1/\sqrt{5}$	$-1/3\sqrt{5}$
$(\gamma K^0|K^0\pi^0)/\sqrt{3}$	$\frac{1}{3}$	$\cdots$	$-2/3\sqrt{5}$
$(\gamma\bar{K}^0|\bar{K}^0\pi^0)/\sqrt{3}$	$-\frac{1}{3}$	$\cdots$	$2/3\sqrt{5}$
$(\gamma K^0|K^+\pi^-)/\sqrt{6}$	$\frac{2}{3}$	$\cdots$	$2/3\sqrt{5}$
$(\gamma\bar{K}^0|\pi^+K^-)/\sqrt{6}$	$\frac{2}{3}$	$\cdots$	$2/3\sqrt{5}$
$(\gamma\pi^0|\pi^+\pi^-)/\sqrt{3}$	$\frac{1}{3}$	$\cdots$	$-2/3\sqrt{5}$
$(\gamma\eta|\pi^+\pi^-)$	-1	$\cdots$	$-2/\sqrt{5}$
$(\gamma K^0|K^+\pi^-)/\sqrt{6}$	$\frac{2}{3}$	$\cdots$	$2/3\sqrt{5}$
$(\gamma\bar{K}^0|\pi^+K^-)/\sqrt{6}$	$\frac{2}{3}$	$\cdots$	$2/3\sqrt{5}$

independent of momentum. Then we have

$$\begin{aligned} f\equiv(K^+|\pi^+)&=\sqrt{2}(K^0|\pi^0)\\ &=(\sqrt{6})(K^0|\eta)=(K_2{}^0|\pi^0)=\sqrt{3}(K_2{}^0|\eta)\,, \end{aligned}\tag{1}$$

where $(P|P')$ denotes the coupling strength of the transition $P\rightarrow P'$.

From Fig. 1 and Eq. (1), one has

$$\begin{aligned} (K^+|\pi^+\pi^0\gamma)=f[(\pi^+|\pi^+\pi^0\gamma)&-(K^+|K^+\pi^0\gamma)\\ &-(K^+|\pi^+K^0\gamma)/\sqrt{2}]D. \end{aligned}\tag{2}$$

For the neutral K^0 decays, we note that

$$K_{10}=(K^0-\bar{K}^0)/\sqrt{2}\,,\quad K_{20}=(K^0+\bar{K}^0)/\sqrt{2}\,.$$

From this it follows that the relations between the decay amplitudes and the diagrams in Fig. 2 are

$$\begin{aligned} (K_1{}^0|\pi^0\pi^0\gamma)&=[(2a)-(2b)+(2c)-(2d)]/\sqrt{2}\,,\\ (K_1{}^0|\pi^+\pi^-\gamma)&=[(2e)-(2f)]/\sqrt{2}\,,\\ (K_2{}^0|\pi^0\pi^0\gamma)&=[(2a)+(2b)+(2c)+(2d)]/\sqrt{2}\,,\\ (K_2{}^0|\pi^+\pi^-\gamma)&=[(2e)+(2f)+\sqrt{2}(2g)+\sqrt{2}(2h)]/\sqrt{2}\,. \end{aligned}\tag{3}$$

Note that the diagrams $K_2{}^0\rightarrow\pi^0\rightarrow\pi^0+\pi^0+\gamma$ and $K_2{}^0\rightarrow\eta\rightarrow\pi^0+\pi^0+\gamma$ do not exist because of charge-conjugation invariance $(\gamma\pi^0|\pi^0\pi^0)=(\gamma\eta|\pi^0\pi^0)=0$.

It may appear that one should not add together the three amplitudes that appear on the right-hand side of Eq. (2) because of the large mass difference between the K and π mesons, but we shall now see that this is not so. Let the four-momenta of the kaon, two pions, and photon be p_K, p_1, p_2, and p_3, respectively. From Fig. (1a), it is evident that conservation of four-momentum requires that the intermediate π^+ has momentum p_K. Similarly, the intermediate K mesons in (1b) and (1c) have the momentum p_1 and p_2, respectively. Thus, in all three diagrams of Fig. 1, a pseudoscalar meson with momentum p_K transforms into two other pseudoscalar mesons with momenta p_1 and p_2 and a photon with momentum p_3. Therefore, the approximation of putting the three amplitudes on an equal footing is valid so long as the interaction that induces the transition $P\rightarrow P+P+\gamma$ is SU_3 invariant and it is

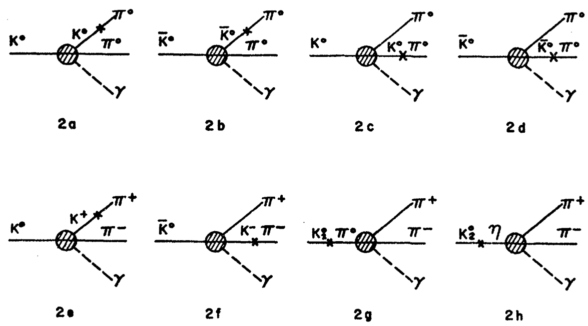

FIG. 2. Diagrams for the decay modes $K_1^0, K_2^0 \to \pi+\pi+\gamma$.

agreed that the amplitudes are compared at equal momenta for corresponding particles. The question of how the symmetry-breaking terms influence the SU_3 predictions is beyond the scope of this paper.

The decay amplitudes, expressed in terms of the channel amplitudes with the aid of Eqs. (1), (2), and (3) and Table I, now are given by

$$\begin{aligned}(K^+|\pi^+\pi^0\gamma) &= (K_1^0|\pi^0\pi^0\gamma)\\ &= (K_1^0|\pi^+\pi^-\gamma) = (K_2^0|\pi^0\pi^0\gamma) = 0,\\ (K_2^0|\pi^+\pi^-\gamma) &= -(4/\sqrt{3})f[A_{10}+(2/\sqrt{5})A_{8a}]D.\end{aligned} \tag{4}$$

The channel amplitudes are functions of $s_i=(p_K-p_i)^2$ $(i=1, 2, 3)$.

The above amplitudes for K mesons are the contribution of the direct term in the boson-pole model. The two pions have the quantum numbers $I=1, J=1, 3, \cdots$ for the channel amplitudes A_{10} and A_{8a}. The $J=1$ are associated with $M1$ transitions.

3. $K\to\pi+\pi+\pi$, $\eta\to\pi+\pi+\pi$

Much as in Sec. 2, the decay amplitudes are obtained by

$$K\to\pi\to\pi+\pi+\pi\,,\quad K\to K+\pi+\pi\to\pi+\pi+\pi\,.$$

The channel amplitudes of various $P+P\to P+P$ are listed in Table II. They are functions of $s_i=(p_K-p_i)^2$, where p_K, p_i are the four-momenta of the kaon and pions, respectively. The index 3 refers to the unlike pion in K^+ decay. The diagrams in the boson-pole model that contribute to $(K^+|\pi^+\pi^+\pi^-)$ and $(K^+|\pi^0\pi^0\pi^+)$ are given in Fig. 3.

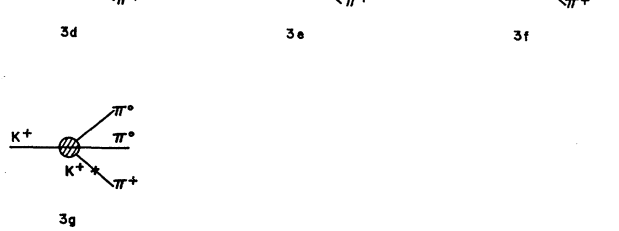

FIG. 3. Diagrams for the decay modes $K^+\to\pi^++\pi^++\pi^-$, $\pi^0+\pi^0+\pi^+$.

From Fig. 3 and Eq. (1) as in Sec. 2, one has

$$\begin{aligned}(K^+|\pi^+\pi^+\pi^-) &= f[(\pi^+|\pi^+\pi^+\pi^-)-(K^+|K^+\pi^+\pi^-)\\ &\qquad -(K^+|\pi^+K^+\pi^-)]D\,,\\ (K^+|\pi^0\pi^0\pi^+) &= f[(\pi^+|\pi^0\pi^0\pi^+)-(K^+|K^0\pi^0\pi^+)/\sqrt{2}\\ &\qquad -(K^+|\pi^0K^0\pi^+)/\sqrt{2}-(K^+|\pi^0\pi^0K^+)]D\,,\end{aligned} \tag{5}$$

where the amplitudes on the right-hand side of Eq. (5)

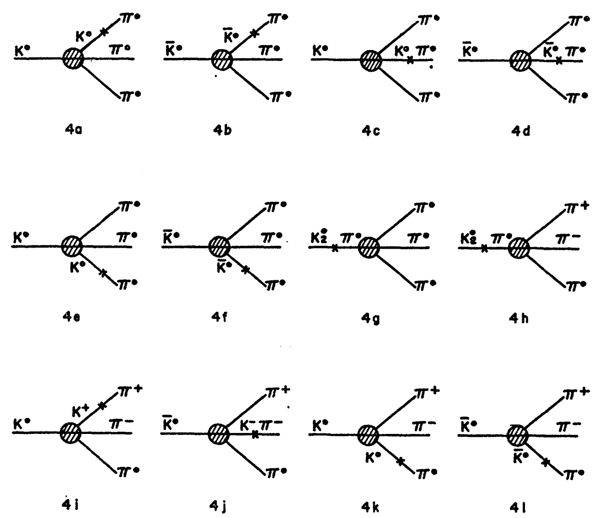

FIG. 4. Diagrams for the decay modes $K_1^0, K_2^0\to\pi+\pi+\pi$.

result from the strong interaction. Much as in the corresponding case in Sec. 2, a pseudoscalar meson with momentum p_K transforms into three other pseudoscalar mesons with momenta p_1, p_2, and p_3 in all the terms.

TABLE II. The $P+P\to P+P$ amplitudes in terms of channel amplitudes. The amplitudes have been multiplied by four.

	B_{27}	B_{8s}	B_1	B_{10}	B_{8a}
$(\pi^+\pi^+\|\pi^+\pi^+)$	4	···	···	···	···
$(K^+\pi^+\|K^+\pi^+)$	2	···	···	2	···
$(\pi^+\pi^-\|\pi^0\pi^0)$	13/10	−4/5	−1/2	···	···
$(K^+\pi^-\|K^0\pi^0)/\sqrt{2}$	3/5	−3/5	···	1/3	−1/3
$(K^+K^-\|\pi^0\pi^0)$	1/10	2/5	−1/2	···	···
$(K^0\pi^0\|K^0\pi^0)$	7/5	3/5	···	5/3	1/3
$(\bar{K}^0\pi^0\|\bar{K}^0\pi^0)$	7/5	3/5	···	5/3	1/3
$(K^0\bar{K}^0\|\pi^0\pi^0)$	−1/10	−2/5	1/2	···	···
$(\pi^0\pi^0\|\pi^0\pi^0)$	27/10	4/5	1/2	···	···
$(\bar{K}^0\pi^0\|\pi^+K^-)/\sqrt{2}$	3/5	−3/5	···	−1/3	1/3
$(K^0\bar{K}^0\|\pi^+\pi^-)$	1/10	2/5	−1/2	−2/3	2/3
$(\eta\pi^0\|\eta\pi^0)$	6/5	4/5	···	2	···
$(\eta\eta\|\pi^0\pi^0)$	3/10	−4/5	1/2	···	···
$(\eta\eta\|\pi^+\pi^-)$	−3/10	4/5	−1/2		

Again we expect that the approximation of treating all the amplitudes on the right-hand side of Eq. (5) on an equal footing, and of adding them, is valid.

There is no ambiguity as to which of the three final bosons should be combined with the initial or intermediate boson so long as the pairing is done consistently.[9] As an example, for the charged K-meson decay modes, the unlike pion [π^- for the $(K^+|\pi^+\pi^+\pi^-)$ and π^+ for $(K^+|\pi^0\pi^0\pi^+)$] can be paired as is indicated in Table II.

Figures 4 and 5 show the diagrams that contribute to the decay modes of the neutral K^0 and η. From these, the decay amplitudes are given as

$$\begin{aligned}
(K_1^0|\pi^0\pi^0\pi^0) &= [(4a)-(4b)+(4c) \\
&\qquad -(4d)+(4e)-(4f)]/\sqrt{2}\,, \\
(K_1^0|\pi^+\pi^-\pi^0) &= [(4i)-(4j)+(4k)-(4l)]/\sqrt{2}\,, \\
(K_2^0|\pi^0\pi^0\pi^0) &= [(4a)+(4b)+(4c) \\
&\qquad +(4d)+(4e)+(4f)+\sqrt{2}(4g)]/\sqrt{2}\,, \\
(K_2^0|\pi^+\pi^-\pi^0) &= [(4i)+(4j)+(4k) \\
&\qquad +(4l)+\sqrt{2}(4h)]/\sqrt{2}\,, \\
(\eta|\pi^0\pi^0\pi^0) &= [(5a)+(5b)+(5c)+(5d)]\,, \\
(\eta|\pi^+\pi^-\pi^0) &= [(5e)+(5f)]\,.
\end{aligned} \tag{6}$$

The diagrams $K_2^0 \to \eta \to \pi^0+\pi^0+\pi^0$ and $K_2^0 \to \eta \to \pi^+ +\pi^- +\pi^0$ do not exist because from G-conjugation invariance it follows that $(\eta\pi^0|\pi^0\pi^0)=(\eta\pi^0|\pi^+\pi^-)=0$.

The decay amplitudes, expressed in terms of the channel amplitudes with the aid of Eqs. (1), (5), (6) and Table II,[10] are given by

$$\begin{aligned}
(K^+|\pi^+\pi^+\pi^-) &= (K^+|\pi^0\pi^0\pi^+) = (K_1^0|\pi^0\pi^0\pi^0) \\
&= (K_1^0|\pi^+\pi^-\pi^0) = (K_2^0|\pi^0\pi^0\pi^0) \\
&= (K_2^0|\pi^+\pi^-\pi^0) = (\eta|\pi^0\pi^0\pi^0) = 0\,, \\
(\eta|\pi^+\pi^-\pi^0) &= (8/5)(\eta|\pi^0)[B_{27}-B_{8s}]\,.
\end{aligned} \tag{7}$$

The bracket $(\eta|\pi^0)$ denotes the weak vertex of the η meson corresponding to $f=(K^+|\pi^+)$ of the K^+ meson.

The three pions in Eq. (7) are in an $I=1$ state, as can be seen from Figs. 3–5. If they are in a totally symmetric state, then

$$B_{27}=B_{8s}=\tfrac{1}{5}B_1\,,\quad B_{10}=B_{8a}=0\,, \tag{8}$$

so that all the $K \to 3\pi$ and $\eta \to 3\pi$ vanish.[11]

[9] The different pairings are related by the crossing matrix.

[10] The relation $(K^+|\pi^0\pi^0\pi^-)=(K_2^0|\pi^+\pi^-\pi^0)=0$ of Eq. (7) is consistent with R. F. Sawyer and K. C. Wali, Nuovo Cimento **17**, 938 (1960). The author thanks K. C. Wali for a discussion.

[11] Equation (8) was obtained during a discussion with K. Itabashi and follows from the requirements $(\pi^+\pi^+|\pi^+\pi^+)=(\pi^+\pi^-|\pi^+\pi^-)$, $(K^+\pi^+|K^+\pi^+)=(K^+\pi^-|K^+\pi^-)$, and $(K^+\pi^0|\pi^+K^0)=(K^+\bar{K}^0|\pi^+\pi^0)$, which hold when the three pions are in a symmetric state.

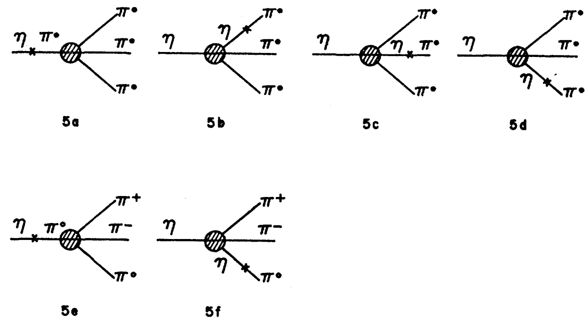

FIG. 5. Diagrams for decay modes $\eta \to \pi+\pi+\pi$.

4. DISCUSSION

The direct term of the decay mode $K \to \pi+\pi+\gamma$ is recorded in Eq. (4). There is no contribution to the direct term of $K^+ \to \pi^+ +\pi^0+\gamma$ within the framework of the boson-pole model combined with unitary symmetry. If experiment establishes that the direct term is needed in any mode except $(K_2^0|\pi^+\pi^-\gamma)$, it would be in contradiction with the present model.

It has been noted[3] that the charged $K \to 3\pi$ decay and the s-wave $\eta \to 3\pi$ decay amplitudes vanish on the basis of an effective interaction that has unitary symmetry, namely

$$4\pi\lambda(\pi\pi+\eta\eta+K\bar{K}+\bar{K}K)^2\,.$$

Our result of Eq. (7) is a rigorous consequence of the boson-pole model and unitary symmetry provided the interaction $P \to P+P+P$ is SU_3 invariant.

The decay amplitudes $K \to 3\pi$ all vanish in the present model.[12] It appears, therefore, that the boson-pole model together with unitary symmetry is untenable for the $K \to 3\pi$ decay modes.

Note added in proof. When the difference between the π mass m and η mass m_0 is taken care of (by the Gell-Mann–Okubo mass formula) in the propagators $1/(M^2-m_0^2)=-3/(M^2-m^2)$, then $(K_2^0|\pi^+\pi^-\gamma)$ also vanishes from Table I and Eq. (3). The author thanks S. Oneda who treated a special case (unpublished) for a discussion.

ACKNOWLEDGMENTS

The author wishes to thank K. Itabashi for many valuable discussions and A. Roberts for his comments that initiated this study.

[12] See, for example, D. Berley, D. Colley, and J. Schultz, Phys. Rev. Letters **10**, 114 (1963).